FINITE PROPAGATION VELOCITY AND LOCAL PERTURBATIONS IN NONLINEAR RELAXATION FILTRATION

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UDC 536.24:517.9

The classic law of nonstationary filtration in a porous medium is Darcy's law, in which it is assumed that the pressure gradient and the velocity instantaneously reach an equilibrium state. Actually "equilibrium" is reached after some delay, and, in order to consider this phenomenon in rheological equations, the velocity v and the pressure p are replaced by $v + \lambda_v\, dv/dt$ and $p + \lambda_p\, dp/dt$, where $\lambda_v > 0$ is the velocity lag time and $\lambda_p > 0$ is the pressure relaxation time. The physical meaning of this substitution is that after instantaneous removal of the pressure drop, the motion does not stop instantaneously, but dies out in time as $\exp(-t/\lambda_v)$. If the motion is stopped instantaneously, the pressure decays in time as $\exp(-t/\lambda_p)$. The breakdown of "equilibrium" relationships between the filtration velocity and the pressure gradient can be explained by relaxation effects, which result from a) the inertia of the fluid and the lag of the velocity from the pressure gradient; b) the relaxation of the pressure and the lag of the pressure gradient from the velocity; c) the complexity of the structure (cracks, caverns, etc.) and the dissipative properties of the porous media; and d) micrononuniformity in the components of the porous media, etc. [1].

The study of filtration processes for non-Newtonian petroleum, polymer solutions, mixtures, emulsions, etc. require explanation of various relaxation effects. In this article the problem of nonstationary filtration in a porous body is examined by using a generalized Darcy's law, in which it is assumed that the equilibrium state between the filtration velocity and the pressure gradient is reached after some delay [1]. Here the pressure relaxation time λ_p is a function of the rate of change of the pressure $\partial p/\partial t$. Conditions are found that give a finite propagation velocity for perturbations and that localize boundary regimes with singularities. An example is constructed which shows that, however small λ_p, the functional dependence $\lambda_p = \lambda(dp/dt)$ leads to an effective localization of the boundary regime with singularities (the LS-regime [2]).

1. If the delay phenomena are considered in the linear approximation, the classical Darcy's law is replaced by

$$v + \lambda_v \frac{\partial v}{\partial t} = -\frac{k}{\mu}\frac{\partial}{\partial x}\left(p + \lambda_p \frac{\partial p}{\partial t}\right), \tag{1.1}$$

where $k > 0$ is the permeability of the porous medium and $\mu > 0$ is the viscosity of the filtering fluid [1, 3]. Viscous elastic Oldroyd fluids of first order [4, 5] satisfy an analogous rheological equation. The relaxation filtration law (1.1) also describes the filtration of a condensed compressive fluid in porous-crack and cavernous media [6].

We assume that the relaxation time λ_p is a function of the rate of change of the pressure; that is, $\lambda_p = \lambda(\partial p/\partial t)$, where $\lambda(u)$ is a continuous function for $u \in R^1$, $\lambda(0) = 0$, $\lambda(u) > 0$, and $\lambda'(u)$ is continuous for $u \neq 0$. It has been established that, however small $\lambda(u)$, effects are observed which are absent in the linear case.

According to the assumptions of elastic theory [3, 6], (1.1) leads to a nonlinear equation for relaxation filtration:

$$\frac{\partial p}{\partial t} + \lambda_v \frac{\partial^2 p}{\partial t^2} = \varkappa \frac{\partial^2}{\partial x^2}\left(p + \lambda\left(\frac{\partial p}{\partial t}\right)\frac{\partial p}{\partial t}\right), \tag{1.2}$$

where $\kappa > 0$ is a known number, which depends on the properties of both the porous medium and the filtering fluid.

Baku. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 102-106, September-October, 1991. Original article submitted January 24, 1990; revision submitted May 3, 1990.

 0021-8944/91/3205-0750$12.50

Equation (1.2) is examined in the region $\Omega \equiv [0; \infty) \times [0; T)$, $0 < T \leq \infty$. There is also a physical meaning to the generalized solution p(x, t) which is continuous and has continuous derivatives $\partial p/\partial t$ and $\frac{\partial}{\partial x}\left(p + \lambda\left(\frac{\partial p}{\partial t}\right)\times\frac{\partial p}{\partial t}\right)$.

2. We will study the question of a finite propagation rate of perturbations of the processes described by (1.2). As is known, Eq. (1.2) does not have such a property for λ = const, and initial or boundary perturbations with an infinite velocity [7].

Let the following condition be fulfilled

$$\int_0^1 \frac{\eta\lambda'(\eta) + \lambda(\eta)}{\left[\int_0^\eta (\xi^2\lambda'(\xi) + \xi\lambda(\xi))\, d\xi\right]^{1/2}} d\eta < \infty. \tag{2.1}$$

In this case the following function makes sense

$$\Phi(u) = \int_0^u \frac{\eta\lambda'(\eta) + \lambda(\eta)}{\left[\int_0^\eta (\xi^2\lambda'(\xi) + \xi\lambda(\xi))\, d\xi\right]^{1/2}} d\eta, \quad u \geqslant 0, \quad \Phi(0) = 0.$$

We will construct a particular self-similar running-wave solution of (1.2)

$$p(x, t) = f(\xi), \quad \xi = \frac{\sqrt{\varkappa}}{\sqrt{\lambda_v}} t - x \geqslant 0.$$

We substitute this function into (1.2) and find

$$f_1' = \varkappa(\lambda(f_1') f_1')'', \tag{2.2}$$

where $f_1 = \sqrt{\kappa/\lambda_v} f$. The nonlinear ordinary differential equation of third order (2.2) is equivalent to the following system of three first-order equations:

$$f_1' = g, \quad \varkappa(\lambda(g) g)' = g_1, \quad g_1' = g. \tag{2.3}$$

From the first and third Eq. (2.3), we obtain

$$\varkappa g d(\lambda(g) g) = g_1 dg_1. \tag{2.4}$$

If we integrate (2.4) and also note that for g = 0 and $g_1 = \kappa[\lambda(g)g]' = 0$, we have

$$g_1 = \sqrt{2\varkappa}\left[\int_0^g \eta d(\eta\lambda(\eta))\right]^{1/2}.$$

By substituting the last equation in the second equation of the system (2.3), we can write

$$\frac{d(\lambda(g) g)}{\left[\int_0^g \eta d(\eta\lambda(\eta))\right]^{1/2}} = \frac{\sqrt{2}}{\sqrt{\varkappa}} d\xi. \tag{2.5}$$

We integrate (2.5) and consider (2.1) to find

$$\Phi(g(\xi)) = \frac{\sqrt{2}}{\sqrt{\varkappa}}(\xi - \xi_0), \quad \xi \geqslant \xi_0.$$

We set $\xi_0 = 0$ and obtain

$$g(\xi) = \Phi^{-1}(\sqrt{2/\varkappa} \cdot \xi), \quad 0 \leqslant \xi < \sqrt{\varkappa/2} \cdot \Phi_*, \quad g(0) = 0$$

where $\Phi_* = \Phi(\infty)$. Finally, from the first equation of the system (2.3), considering that $f(\xi) = \sqrt{\lambda_v/\kappa} \cdot f_1(\xi)$, and f(0) = 0, we find

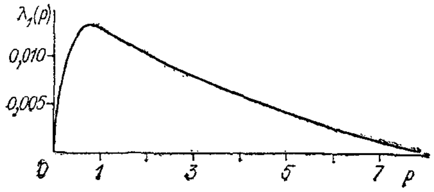

Fig. 1

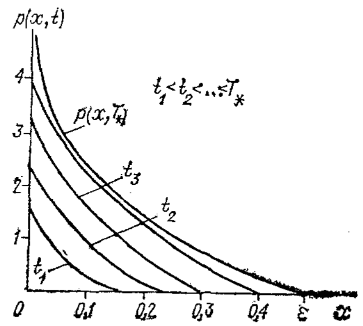

Fig. 2

$$f(\xi)=\sqrt{\lambda_v/\varkappa}\int\limits_0^{\xi}\Phi^{-1}(\sqrt{2/\varkappa}\cdot\eta)\,d\eta,\quad 0\leqslant\xi<\sqrt{\varkappa/2}\cdot\Phi_*,\quad f(0)=0.$$

We set the function $f(\xi)$ to zero in the region $\{\xi < 0\}$. Because $\Phi^{-1}(0) = 0$, then $f(\xi)$ and $f'(\xi) = \sqrt{\lambda_V/\kappa}\cdot\Phi^{-1}(\sqrt{2/\kappa}\cdot\xi)$ are continuous. If

$$[\lambda(\sqrt{\varkappa/\lambda_v}\cdot f')f']'=\{\sqrt{\lambda_v/\varkappa}\cdot\lambda[\Phi^{-1}(\sqrt{2/\varkappa}\cdot\xi)]\Phi^{-1}(\sqrt{2/\varkappa}\cdot\xi)\}=$$
$$=\frac{\sqrt{2\lambda_v}}{\varkappa}\left[\int\limits_0^{\Phi^{-1}(\sqrt{2/\varkappa}\cdot\xi)}(\eta^2\lambda'(\eta)+\eta\lambda(\eta))\,d\eta\right]^{1/2},$$

then the function $[\lambda(\sqrt{\kappa/\lambda_V}\cdot f')f']'$ is also continuous for $0 \leq \xi < \sqrt{\kappa/2}\cdot\Phi_*$ and remains so after continuation into the region $\{\xi < 0\}$.

Thus, we construct the generalized solution to (1.2):

$$p(x,t)=\begin{cases}\sqrt{\lambda_v/\varkappa}\int\limits_0^{\xi}\Phi^{-1}(\sqrt{2/\varkappa}\cdot\eta)\,d\eta, & 0\leqslant x\leqslant\sqrt{\varkappa/\lambda_v}\cdot t,\\ 0, & x\geqslant\sqrt{\varkappa/\lambda_v}\cdot t,\end{cases}\tag{2.6}$$

where $\xi = \sqrt{\kappa/\lambda_V}\cdot t - x$. It exists over a time $0 \leq t < T_* = \sqrt{\lambda_V/2}\cdot\Phi_* \leq \infty$ and satisfies the following initial conditions on the boundary in Ω:

$$p(x,0)=0,\ \partial p(x,0)/\partial t=0,\ 0\leqslant x<\infty;\tag{2.7}$$

$$p(0,t)=\sqrt{\lambda_v/\varkappa}\int\limits_0^{\sqrt{\varkappa/\lambda_v}\cdot t}\Phi^{-1}(\sqrt{2/\varkappa}\cdot\eta)\,d\eta,\quad p(\infty,t)=0,\quad 0\leqslant t<T_*.\tag{2.8}$$

Thus, Eq. (1.2) has a solution which is finite along x for every $t \in [0; T_*)$; $p(x, t) = 0$ for $x \geq \sqrt{\kappa/\lambda_V}\cdot t$.

<u>Example 1.</u> Let $\lambda(u) = |u|^n$. It is easy to verify that the condition (2.1) is fulfilled for $n > 0$, and the solution (2.6) takes the form

$$p(x,t)=\begin{cases}\sqrt{\dfrac{2\lambda_v(n+1)}{n+2}}\left[\dfrac{n(\sqrt{\varkappa/\lambda_v}\cdot t-x)}{\sqrt{2\varkappa(n+1)(n+2)}}\right]^{(n+2)/n}, & 0\leqslant x\leqslant\sqrt{\varkappa/\lambda_v}\cdot t,\\ 0, & x\geqslant\sqrt{\varkappa/\lambda_v}\cdot t,\quad 0\leqslant t<\infty\end{cases}$$

for the initial conditions (2.7) and the boundary conditions

$$p(0,t)=\sqrt{\frac{2\lambda_v(n+1)}{n+2}}\left[\frac{nt}{\sqrt{2\lambda_v(n+1)(n+2)}}\right]^{(n+2)/n},\quad p(\infty,t)=0,$$
$$0\leqslant t<\infty.$$

For $t > 0$, $p(x, t)$ has a bounded carrier $[0; \sqrt{\kappa/\lambda_v}\cdot t]$, in which there is a convex function in x.

3. Let $\Phi_* < \infty$ and

$$\int_0^{\Phi_*} \Phi^{-1}(\eta)\, d\eta = \infty. \tag{3.1}$$

In this case the solution (2.6) exists along a bounded section to the time $[0; T_*]$ and, as can be seen from (2.8), $p(0, t) \to \infty$ for $t \to T_* - 0$; that is, we observe a boundary regime with singularities. However, in spite of that, perturbations are localized on a bounded section $[0; x_*]$, where $x_* = \sqrt{\kappa/2}\cdot\Phi_* < \infty$. Moreover, a limiting pressure distribution exists at the moment of the singularity T_*:

$$p(x, T_*) = \begin{cases} \sqrt{\lambda_v/2} \displaystyle\int_0^{\Phi_* - \sqrt{2/\kappa}\cdot x} \Phi^{-1}(\eta)\, d\eta, & 0 < x \leqslant x_*, \\ 0, \quad x \geqslant x_* \end{cases}$$

and $p(x, T_*) < \infty$ for $x > 0$. In the terminology of [2], this means that there is an LS regime with a singularity.

Example 2. Let $\kappa\lambda(u) = \lambda_\varepsilon(u)$, where

$$\lambda_\varepsilon(u) = \frac{\varepsilon^2}{1+\varepsilon u} - \frac{\varepsilon \ln(1+\varepsilon u)}{u(1+\varepsilon u)} - \frac{\varepsilon^3 u}{2(1+\varepsilon u)^2}, \quad u > 0, \quad \lambda_\varepsilon(0) = 0,$$

where $\varepsilon > 0$ is an arbitrary number. Figure 1 shows the function λ_ε for $\varepsilon = 1$. In this case Eq. (1.2) has a solution

$$p(x, t) = \begin{cases} \dfrac{\sqrt{\lambda_v}}{\sqrt{\kappa}} \left\{ -\ln\left[1 - \dfrac{1}{\varepsilon}\left(\dfrac{\sqrt{\kappa}}{\sqrt{\lambda_v}} t - x\right)\right] - \dfrac{\sqrt{\kappa}}{\varepsilon\sqrt{\lambda_v}} t + \dfrac{x}{\varepsilon} \right\}, \\ \qquad\qquad 0 \leqslant x \leqslant \sqrt{\kappa/\lambda_v}\cdot t, \\ 0, \quad x \geqslant \sqrt{\kappa/\lambda_v}\cdot t \end{cases} \tag{3.2}$$

for $0 \le t < T_* = \sqrt{\lambda_v/\kappa}\cdot\varepsilon$. A graph of the function $p(x, t)$ at various moments in time is shown in Fig. 2. The solution (3.2) satisfies the initial conditions (2.7) and the following boundary conditions:

$$p(0, t) = \sqrt{\lambda_v/\kappa}\left\{-\ln\left[1 - \frac{\sqrt{\kappa/\lambda_v}}{\varepsilon} t\right] - \frac{\sqrt{\kappa/\lambda_v}}{\varepsilon} t\right\}, \quad 0 \leqslant t < T_*,$$
$$p(\infty, t) = 0, \quad 0 \leqslant t < T_*.$$

As can be seen $p(0, t) \to \infty$ for $t \to T_* - 0$ and at the time of the singularity, the wave penetrates to a finite depth $x_* = \varepsilon$, and $p(x, t) = 0$ for $x \ge x_*$ and $0 \le t \le T_*$. Except for the point $x = 0$, the solution $p(x, t)$ is uniform for $t \in [0; T_*]$, and is bounded by the limiting curve $p(x, T_*)$ (Fig. 2):

$$p(x, t) \leqslant p(x, T_*) = \begin{cases} \sqrt{\lambda_v/\kappa}\left[-\ln\left(\dfrac{x}{\varepsilon}\right) - 1 + \dfrac{x}{\varepsilon}\right], & 0 \leqslant x \leqslant \varepsilon, \\ 0, \quad x \geqslant \varepsilon. \end{cases}$$

It is not difficult to verify that $\lim_{\varepsilon\to 0} \lambda_\varepsilon(u) = 0$ for any $u \ge 0$. Because of the choice of ε, Fig. 1 shows that the function λ can be set arbitrarily small, but in spite of that, the function $\lambda(\partial p/\partial t)$ leads to the effect of a localization of the boundary regime with singularities.

The author thanks A. Kh. Mirzadzhanzade for posing the problem and discussing the results.

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FEATURES OF THE PRESSURE–ATTENUATION CURVE IN RELAXATION FILTRATION OF A FLUID

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Laboratory experiments have shown that, for fluid filtration processes with a characteristic fluctuation time of $\sim 10^3$ sec, theoretical predictions based on a model of the elastic regime can differ from observed quantities by an order of magnitude [1-3]. Therefore, in describing rapidly varying fluid filtration phenomena, the classic elastic equations [4, 5] must be avoided, and equations from the relaxation theory of filtration [6, 7] must be used instead, in particular, for the initial section of the pressure–attenuation curve. In earlier approximate formulas for the pressure–attenuation curve, the relaxation kernel had a somewhat special form [6]. The most general case [6] corresponds to a vibrating Fourier-type relaxation kernel in the form of a ratio of two second-order polynomials. In this work exact results are found for the initial section of the pressure–attenuation curve for an arbitrary kernel which is consistent with physical and thermodynamic requirements.

1. We examine a homogeneous porous medium which is saturated with fluid. Isothermal processes are studied in which the fluid density ρ differs only slightly from some fixed value ρ_0; therefore a linear expression can be used for the pressure

$$p = p_0 + E(\rho - \rho_0)/\rho_0. \tag{1.1}$$

In the relaxation theory of filtration [6, 7], Darcy's law is generalized as follows:

$$\mathbf{u}(t_0, r) = -k\mu^{-1} \int_{-\infty}^{+\infty} K(t_0 - t)\nabla G(t, \mathbf{r})\,dt, \quad G = p + \rho\varphi. \tag{1.2}$$

Here $\mathbf{u}$ is the filtration velocity; k is the permeability; φ is the gravitational potential; and μ is the viscosity, which will be considered constant. The kernel K = K(t), which does not depend on the spatial coordinates, characterizes the internal relaxation processes in the system of the porous medium and the fluid. The function K = K(t) satisfies a series of conditions which follow from physical and thermodynamic considerations [2]:

Moscow. Translated from Zhurnal Prikladnoi' Mekhaniki i Tekhnicheskoi' Fiziki, No. 5, pp. 106-111, September-October, 1991. Original article submitted December 12, 1989; revision submitted May 15, 1990.

 0021-8944/91/3205-0754$12.50